Effect of Parallel Transport Currents on the D-wave Josephson junction

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Abstract. In this paper, the nonlocal mixing of coherent current states in d-wave superconducting banks is investigated. The superconducting banks are connected via a ballistic point contact. The banks have a mis-orientation and a phase difference. Furthermore, they are subjected to a tangential transport current along the ab-plane of d-wave crystals and parallel with interface between superconductors. The effects of mis-orientation and external transport current on the current-phase relations and current distributions are subjects of this paper. It observed that, at values of phase difference close to 0, π and 2π the current distribution may have a vortex-like form in vicinity of the point contact. The current distribution of this junction between d-wave superconductors is totally different from the junction between s-wave superconductors. As the interesting results, spontaneous and Josephson currents are observed for the case of $\phi = 0$.

1. Introduction

The weak link between d-wave superconductors is a long studied problem theoretically [1, 2, 3, 4, 5, 6, 7, 8, 9]. A theoretical investigation of the total transparent Josephson junction between two d-wave superconductors has been done in Ref.[1]. Using the quasiclassical approach, a d-wave Josephson Junction with low-transparent interface has been studied in Ref.[2]. Anisotropic and unconventional pairing symmetry has been considered for $d - I - d$ systems and zero energy states (ZES) as the result of sign change of the order parameter have been observed in paper [3]. A spontaneous current parallel to the interface between d-wave superconductors has been found in paper[4]. The junction between current-carrying states of d-wave superconductors, has been investigated in Ref.[4]. Authors of paper [4] by numerical self-consistent calculations shows that, the supercurrent parallel to the junction may flow in the direction opposite to the current direction at the superconducting banks. Effect of transparency of interface of d-wave Josephson junction was studied in Ref.[5]. In Ref.[6], effects of transparency and mis-orientation of two $d-$wave crystals have been investigated analytically. Using the Bogoliobov-De Gennes equations ZES as the origin of zero bias conductance peak (ZBCP) were studied in Refs.[7, 8]. ZES are introduced as the fingerprint of unconventional pairing symmetry in[7, 8]. In Ref.[9], a special geometry of $d-$wave superconducting layer as a weak link has been investigated and the π Josephson junction has been observed. Also because of high critical temperature of d-wave superconductors (and cheap production technology), many experimental works about the d-wave weak link have been done in the las two decades [10, 11, 12, 13, 14, 15, 17]. A complete review of these experiments has been presented in a review paper [10]. A phase sensitive experiment (phase of the superconducting order parameter) has been presented by authors of [11] for determination of the symmetry of order parameter in high T_c cuperate superconductors. Using the phase interference experiments in the Josephson junctions, $d-$wave pairing symmetry in the cuperate superconductors has been observed in Refs.[11, 12, 13]. A nonsinusoidal form of current-phase diagram has been observed in [14], experimentally. Authors of Ref.[15] have measured the current-phase relationship of symmetric grain boundary weak link and observed that, temperature controlled sign change of the first harmonic of the Josephson current $(I(\phi) = I_1 \sin\phi + I_2 \sin 2\phi + \cdots)$. Because of competition between the first and second harmonics of Josephson current, a nonmonotonic temperature dependence of the critical current has been reported in [15]. An experimental investigation of a Josephson junction between $d-$wave superconductors has been done and effect of insulator between them has been studied in [17]. They observed $0 - \pi$ transitions by reducing the width of insulator in $d - I - d$ Josephson junction. From the other hand, it is well-known that, non-locality and Josephson effect are coexisting. Charged particles orbiting around a magnetic flux are influenced by a magnetic flux as a phase difference although, the region including flux is forbidden for charged particles. This phase as the Aharonov-Bohm phase is a demonstration of non-locality of quantum mechanics. While the supercurrent in a superconducting bulk

depends on the phase gradient locally[18], $\mathbf{j}(\mathbf{r}) \propto \nabla\varphi(\mathbf{r})$, the Josephson supercurrent depends on phase difference non-locally[19], $\mathbf{j}(\varphi_2 - \varphi_1) \propto \sin(\varphi_2 - \varphi_1) \propto (\varphi_2 - \varphi_1)$. The interplay between local supercurrent states and non-local phase difference between the superconducting bulks is an interesting problem. Local supercurrent states are introduced by superfluid velocity of Cooper pairs. The, non-locality of Josephson current in the point contacts and the effect of super-fluid velocity on the current states in narrow films and wires have been studied in Ref.[20]. An anomalous periodic behavior in terms of magnetic flux has been observed in Ref.[20]. This anomalous property is demonstrated as a result of a non-locality of supercurrent in the Josephson junction[20]. In addition, experimental results of Ref.[20] have been confirmed in analytical calculations of Ref.[21]. The dynamical Josephson junction between s-wave superconductors has been investigated in Ref.[22]. They studied the quantum interference between right and left s-wave superconductors, in which parallel transport currents are flowed. The existence of two anti-symmetric vortex-like currents near the contact and at $\phi \simeq \pi$ as a new phenomenon was reported in Ref.[22]. The authors of Ref.[22] found that the total current is not the vector sum of Josephson and the transport currents because of a new term in the current. This term is called the "interference" current and can be a "parallel Josephson current". In the Ref.[22], effect of reflection at the interface between $s-$wave superconductors has been investigated analytically and numerically.

In this paper, a planar weak link between two d-wave superconductors with a phase difference between order parameters is investigated. The ab-planes of two superconductors have a mis-orientation and c axis of two crystals are parallel to the interface between d-wave superconductors. In the center of the interface we create an ideal transparent thin slit with length L and width a. Interference between wave functions of the left and right superconductors occurs through the slit. The remainder part of interface is an ideal insulator and is impenetrable for Cooper pairs. Also, two transport supercurrents at the ab-planes are flowing parallel to the insulator and the contact plane (see Fig.1). The Josephson current from one of the bulks to another, is a result of the interference between states with phase difference ϕ as is predicted in Ref.[19]. The contact scales (thin slit), length L and width a, are much larger than the Fermi wavelength and smaller than coherence length of superconductivity. Furthermore, these scales are small as compared with the mean free path of quasi-particles. Therefore the quasi-classical approximation for the ballistic point contact can be used. The Eilenberger equations for this system are solved and Green functions are obtained. The effects of mis-orientation and phase difference between order parameters and super-fluid velocity on the current distributions and current-phase graphs are investigated in this paper.

The organization of the rest of this paper is as follows. In Sec.2 the quasiclassical equations for Green functions are presented. The obtained formulas for the Green functions are used to analyze a current state in the ballistic point contact. Also the effects of transport current and mis-orientation on the current distribution at the contact
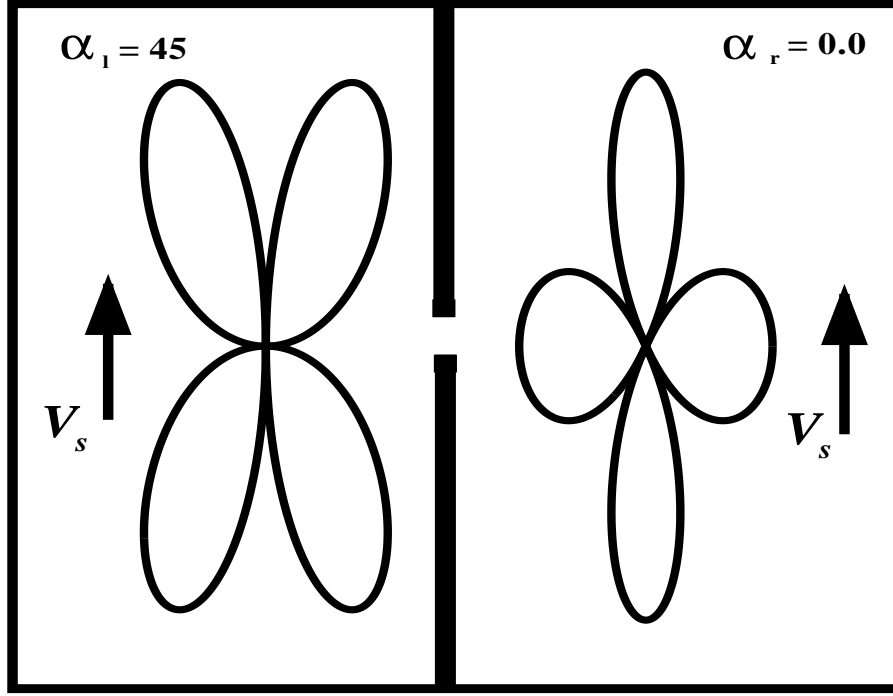
Figure 1. Model of the contact in the insulating partition, along the $\hat{\mathbf{y}}$, between two mis-oriented d-wave superconducting bulks with transport supercurrent on the banks. The plane of paper is ab-plane of d-wave superconductors. In the d-wave superconductors like $YBaCuO$ the ab-plane is the plane of CuO.

plane are investigated. In Sec.3 the results of simulation for current distribution in vicinity of the contact will be investigated. An analytical investigation of system near the critical temperature will be done in Sec.4. The paper will be finished with some conclusions in Sec.5.

2. Formalism and Basic Equations

The Eilenberger equations for the ξ-integrated Green's functions are used to describe the coherent current states in a superconducting ballistic micro-structures [23]:

$$\mathbf{v}_F \cdot \frac{\partial}{\partial \mathbf{r}} \widehat{G}_\omega(\mathbf{v}_F, \mathbf{r}) + [\omega \widehat{\tau}_3 + \widehat{\Delta}(\mathbf{v}_F, \mathbf{r}), \widehat{G}_\omega(\mathbf{v}_F, \mathbf{r})] = 0 \tag{1}$$

where

$$\widehat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^\dagger & 0 \end{pmatrix}, \quad \widehat{G}_\omega(\mathbf{v}_F, \mathbf{r}) = \begin{pmatrix} g_\omega & f_\omega \\ f_\omega^\dagger & -g_\omega \end{pmatrix} \tag{2}$$
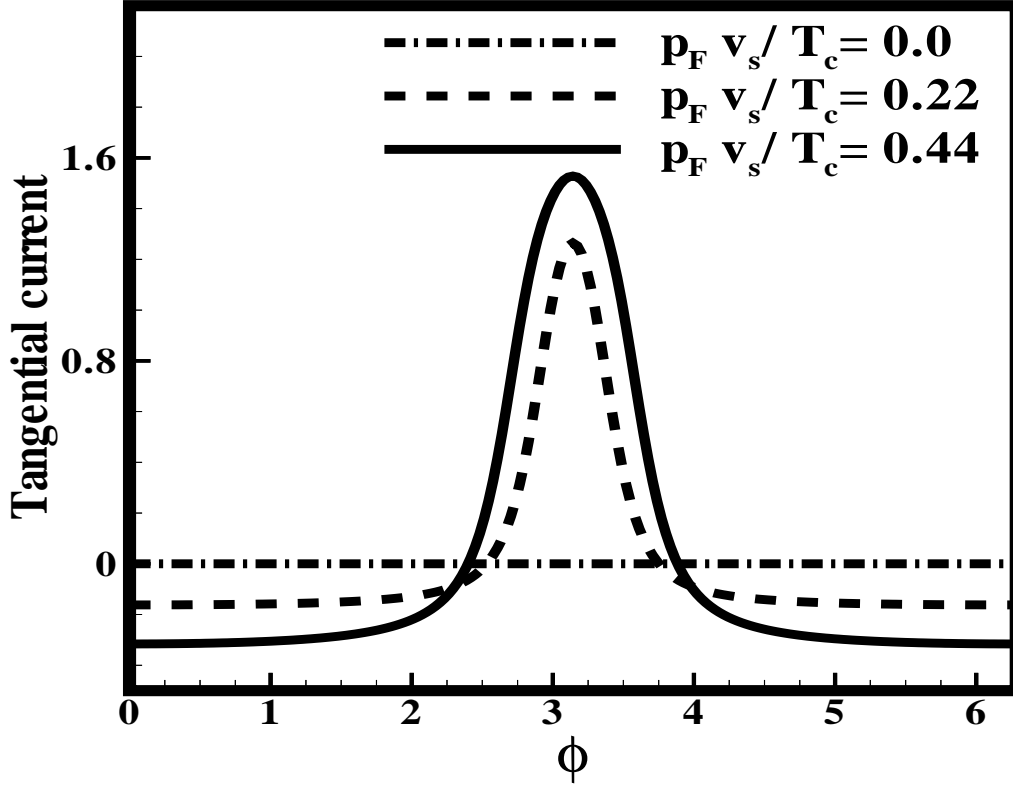
Figure 2. Tangential current j_y versus ϕ for $T/T_c = 0.1$, $\alpha_l = \alpha_r = 0$ in the units of $j_0 = 4\pi e N(0) v_F T$. This is like the case of junction between conventional superconductors[22]

Δ is the superconducting order parameter, $\widehat{\tau}_3$ is the Pauli matrix, and $\widehat{G}_\omega(\mathbf{v}_F, \mathbf{r})$ is the matrix Green function which depends on the electron velocity on the Fermi surface $\mathbf{v}_F$, the coordinate $\mathbf{r}$ and Matsubara frequency $\omega = (2n+1)\pi T$, with n and T being an integer number and temperature respectively. Also the normalization condition

$$g_\omega = \sqrt{1 - f_\omega f_\omega^\dagger} \tag{3}$$

with $f_\omega^\dagger$ being time-reversal counterpart of f_ω should be satisfied by solutions of the Eilenberger equations. In general, Δ depends on the direction of $\mathbf{v}_F$ and $\mathbf{r}$ and it can be determined by the self-consistent equation

$$\Delta(\mathbf{v}_F, \mathbf{r}) = 2\pi N(0) T \sum_{\omega > 0} \langle V(\mathbf{v}_F, \mathbf{v}_F') f_\omega(\mathbf{v}_F', \mathbf{r}) \rangle_{\mathbf{v}_F'} \tag{4}$$

and the current density by

$$\mathbf{j}(\mathbf{r}) = -4\pi i e N(0) T \sum_{\omega} \langle \mathbf{v}_F g_\omega(\mathbf{v}_F, \mathbf{r}) \rangle_{\mathbf{v}_F} \tag{5}$$

respectively, where $V(\mathbf{v}_F, \mathbf{v}_F')$ is the interaction potential, $N(0)$ is the 2D density of states at the Fermi surface for each spin projection and $\langle ... \rangle$ is the averaging over
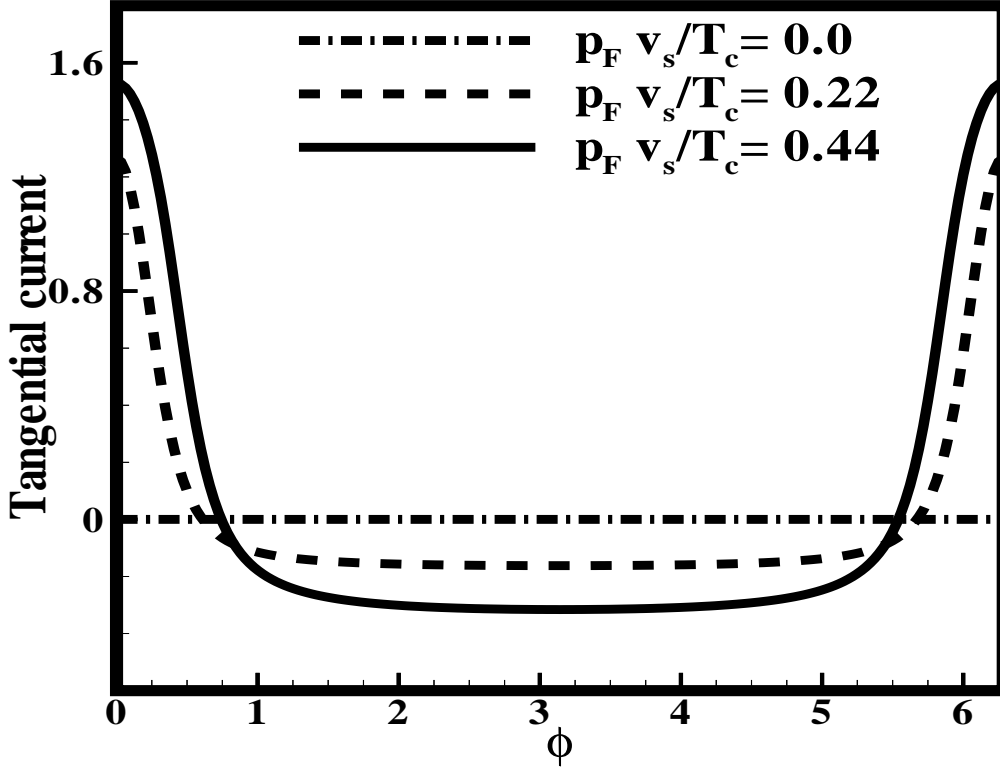
Figure 3. Tangential current j_y versus phase ϕ for $T/T_c = 0.1$, $\alpha_l = 0$ and $\alpha_r = \frac{\pi}{2}$.

directions of $\mathbf{v}_F$. Solution of the matrix equation (1) together with self-consistency gap equation (4), Maxwell equation for superfluid velocity and normalization condition determines the current $\mathbf{j}(\mathbf{r})$ in the system. The thickness of d-wave superconductors are assumed to be smaller than coherence length $\xi_0 = \frac{\hbar v_F}{\pi \Delta}$. Thus the spatial distributions of $\Delta(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$ depend only on the coordinates in the plane of the film and Eilenberger equations (1) reduce to 2D equations.

Also equations (1) for Green functions $\widehat{G}_\omega(\mathbf{r}, \mathbf{v}_F)$ have to be supplemented by the condition of specular reflection at the region $(x = 0, |y| \geq a)$ and continuity of solutions across the point contact $(x = 0, |y| \leq a)$. Far from the contact the Green functions should be coincident with the bulk solutions and the current should be homogeneous transport current along the y-axis.

In this formalism, the current has to be determined by a self-consistent gap equation (4) together with the Maxwell equation for superfluid velocity (Ampere law $\frac{d^2 A_y(x)}{dx^2} = -\mu_0 J_y^{tot}(x)$ with μ_0 for free space and $v_s = -\frac{e}{mc} A_y(x)$ for superfluid velocity as in paper [5, 24]). While for simplicity, the self-consistency of order parameter is ignored and a step function is considered for spatial dependence and we do not consider effect of current distribution on the superfluid velocity. We believe that as in the papers [25, 26], the

self-consistent investigation of d-wave Josephson junction does not show a qualitative different from the non-self-consistent results.

For Δ and $\mathbf{v_s}$ being constants at each half plane an analytical solution for Eilenberger equations can be found by the method of integration along the quasi-classical trajectories of quasi-particles. In any point, $\mathbf{r} = (x, y)$, all ballistic trajectories can be categorized as transit and non-transit trajectories.

For the transit trajectories the Green functions satisfy continuity at the contact and the non-transit trajectories satisfy the specular reflection condition at the partition, $(x = 0, |y| \geq a)$. Also, all transit and non-transit trajectories should satisfy the boundary conditions in the left and right bulks. Making use of solutions of the Eilenberger equations, the following expression is obtained for current at the slit:

$$\mathbf{j}(x = 0, |y| < a, \phi, \mathbf{v_s}, \alpha_l, \alpha_r) = 4\pi e N(0) v_F T \sum_{\omega > 0} \left\langle \widehat{\mathbf{v}} \frac{\widetilde{\omega}(\Omega_l + \Omega_r) - i\eta\Delta_l\Delta_r \sin\phi}{\Omega_l\Omega_r + \widetilde{\omega}^2 + \Delta_l\Delta_r \cos\phi} \right\rangle_{\widehat{\mathbf{v}}} \quad (6)$$

where, $\Delta_{l,r} = \Delta_0 \cos(2(\theta - \alpha_{l,r}))$ for $d_{x^2 - y^2}$ symmetry $\Omega_{l,r} = \sqrt{\widetilde{\omega}^2 + \Delta_{l,r}^2}$, $\widetilde{\omega} = \omega + i\mathbf{p}_F\mathbf{v_s}$ with ω being Matsubara frequency and $\mathbf{v_s}$ is super-fluid velocity, $\widehat{\mathbf{v}} = \mathbf{v}_F/v_F$ is the unit vector, $\eta = sign(v_x)$. In this non-stationary Josephson junction, $\mathbf{v}_s \neq 0$, the current has both $\mathbf{j}_x$ and $\mathbf{j}_y$ components. We define the Josephson current, external transport current, spontaneous current and interference current as:

$$\mathbf{j}_{Josephson} = \mathbf{j}(\phi, \mathbf{v_s}, \alpha_l, \alpha_r)_{\widehat{x}} \quad (7)$$

$$\mathbf{j}_{Transport} = \mathbf{j}(\phi = 0, \mathbf{v_s} \neq 0, \alpha_l = \alpha_r = 0)_{\widehat{y}} = \mathbf{j}_{Bulk}, \quad (8)$$

$$\mathbf{j}_{Spontaneous} = \mathbf{j}(\phi \neq 0, \mathbf{v_s} = 0, \alpha_l \neq 0, \alpha_r \neq 0)_{\widehat{y}} \quad (9)$$

$$\mathbf{j}_{JT} = \mathbf{j}(\phi, \mathbf{v_s}, \alpha_l, \alpha_r)_{\widehat{y}} - \mathbf{j}_{Spontaneous} - \mathbf{j}_{Transport} \quad (10)$$

respectively. The Josephson current, $j_J = j_x$, is normal to the interface between superconductors as was considered by B. D. Josephson in [19] and parallel component of current $\mathbf{j}_y$ consists of three terms of current, an external transport current, spontaneous current and "interference" current. The new current term, "interference" current, depends on the super-fluid velocity, orientations with respect to the interface and phase difference between order parameters. This term of current, $\mathbf{j}_{JT}$, is completely different from the transport current on the banks $\mathbf{j}_T$.

Thus in addition to the spontaneous current for the stationary d-wave Josephson junction that is investigated in Ref.[6] and transport current, we observe another current parallel to the interface ($\mathbf{j}_{JT}$). In particular, at $\phi \simeq \pi$ it may go into the opposite direction to the external transport current on the banks (depending on the orientations). This sign reversal of tangential supercurrent which is origin of vortex-like currents near the orifice has been observed already in Ref.[4] for the case of $d-$wave junction. At two sides of vortex-like currents, currents are flowing parallel and antiparallel to the external supercurrent at the superconducting banks. In the Ref.[4] a superconductor-normal-superconductor trajectory for particles has been considered while there is one superconductor coupled the normal metal. So, because of one superconductor in the

structure of Ref.[4] it was impossible to consider the phase difference. In the case of junction between $s-$wave superconductors in Ref.[22], it was observed that sign reversal can be seen only for $\phi \simeq \pi$. So, for $s-$wave counterpart of setup of paper [4] it is impossible to see the sign reversal because phase difference ϕ does not meaning for system including one superconductor. While in our calculations in this paper we have observed that, this sign reversal and vortex appearance can be seen at $\phi = 0$, depending on mis-orientation. For the case of $d-$wave in paper of [4] in the absence of phase difference, a sign reversal of current has been observed as well as our results for suitable mis-orientation of $ab-$planes($\alpha_l = 0, \alpha_r = \frac{\pi}{4}$ or planes (1,0,0),(1,1,0)). The planes $\alpha_l = 0$ and $\alpha_r = \frac{\pi}{4}$ are corresponding to planes (100) and (111) respectively.
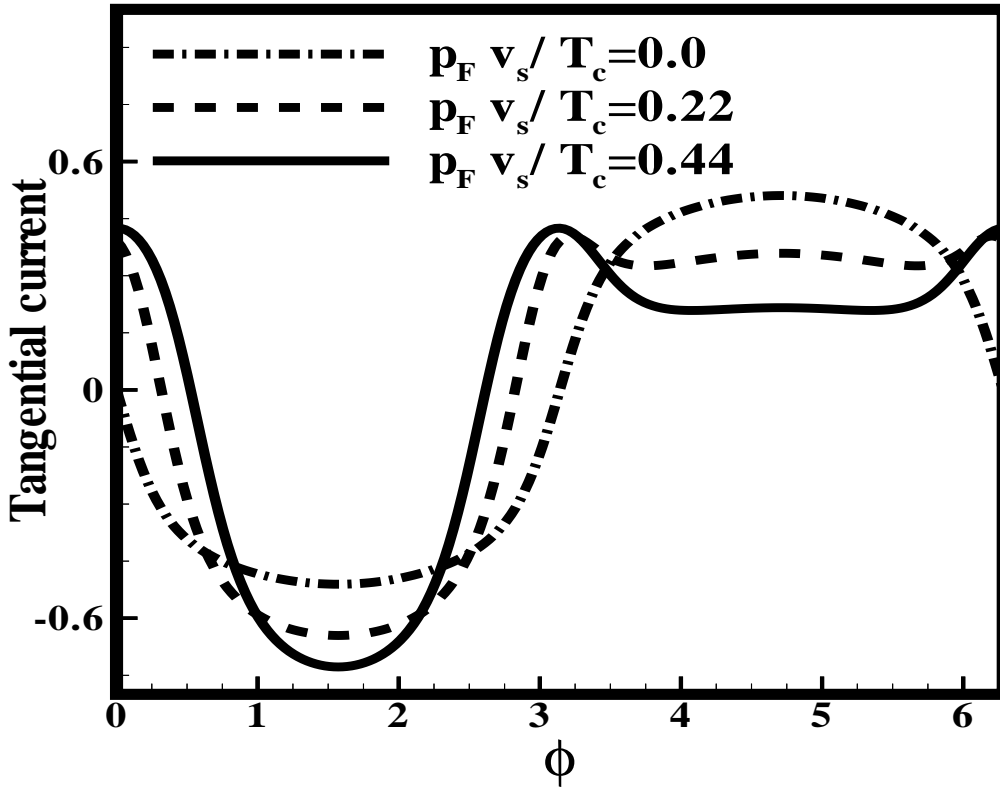

Figure 4. Tangential current j_y versus phase ϕ for $T/T_c = 0.1$, $\alpha_l = 0$ and $\alpha_r = \frac{\pi}{4}$.

3. discussion

The effects of mis-orientation and super-fluid velocity on the current-phase relation and current distribution are investigated numerically and the results are following.
1) The vortex-like currents appear at values of ϕ when the parallel current is maximum and positive while the external transport current is negative (Figs.2,3,4 and 5).
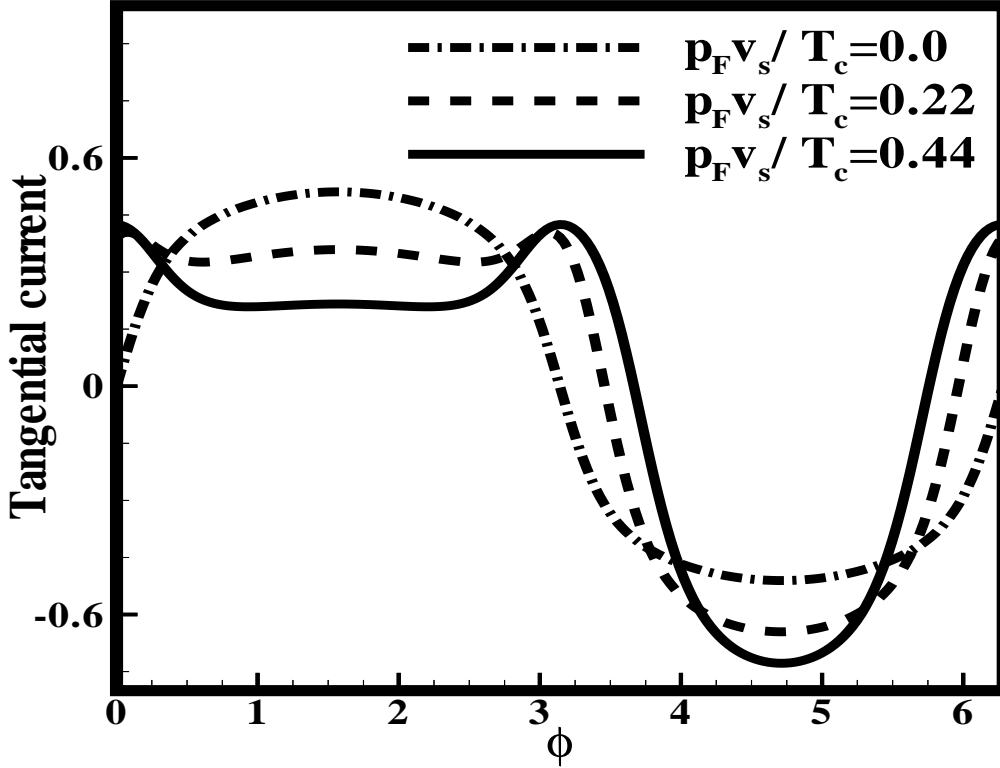
Figure 5. Tangential current j_y versus phase ϕ for $T/T_c = 0.1$, $\alpha_l = 0$ and $\alpha_r = -\frac{\pi}{4}$.

2) At $\alpha_l = \alpha_r = 0$ and arbitrary ϕ, the current distributions and current-phase graphs are identical with the s-wave which is investigated in Ref.[22]. For example at $\alpha_l = \alpha_r = 0$ and $\phi = \pi$ two anti-symmetric vortex-like currents are observed and their common axis is normal to the interface (Fig.6).

3) At $\alpha_l = 0$, $\alpha_r = \pm\frac{\pi}{4}$ and $\phi = \pi$, two vortex-like currents are observed. Their common axis is rotated as much as $\mp\frac{\pi}{4}$ (Fig.8 and Fig.9). This case occurs for $\alpha_l = 0$, $\phi = \pi$ and arbitrary α_r with the rotated axis rotating as much as $(-\alpha_r)$.

4) The appearance of vortex-like currents can be controlled by mis-orientation. For example, at $\phi = 0$ and $\alpha_l = \alpha_r = 0$, we cannot observe the vortex-like currents as we cannot observe in s-wave junction (Fig.7). However, for $\phi = 0$, $\alpha_{l,r} = 0$, $\alpha_{r,l} = \frac{\pi}{2}$ the vortex-like currents appear since from (6) and for $d_{x^2-y^2}$ symmetry we have :

$$\mathbf{j}(\mathbf{r}, \phi = \pi, \mathbf{v}_s, \alpha_l, \alpha_r) = \mathbf{j}(\mathbf{r}, \phi = 0, \mathbf{v}_s, \alpha_l, \alpha_r + \frac{\pi}{2}). \tag{11}$$

5) In Figs.3,4,5 it is observed that for $\phi = 0$ and consequently without any external magnetic flux, the interference between coherent current-states can occur and the vortex-like currents can be observed because mis-orientation plays the role of the phase difference in Eq.(11).
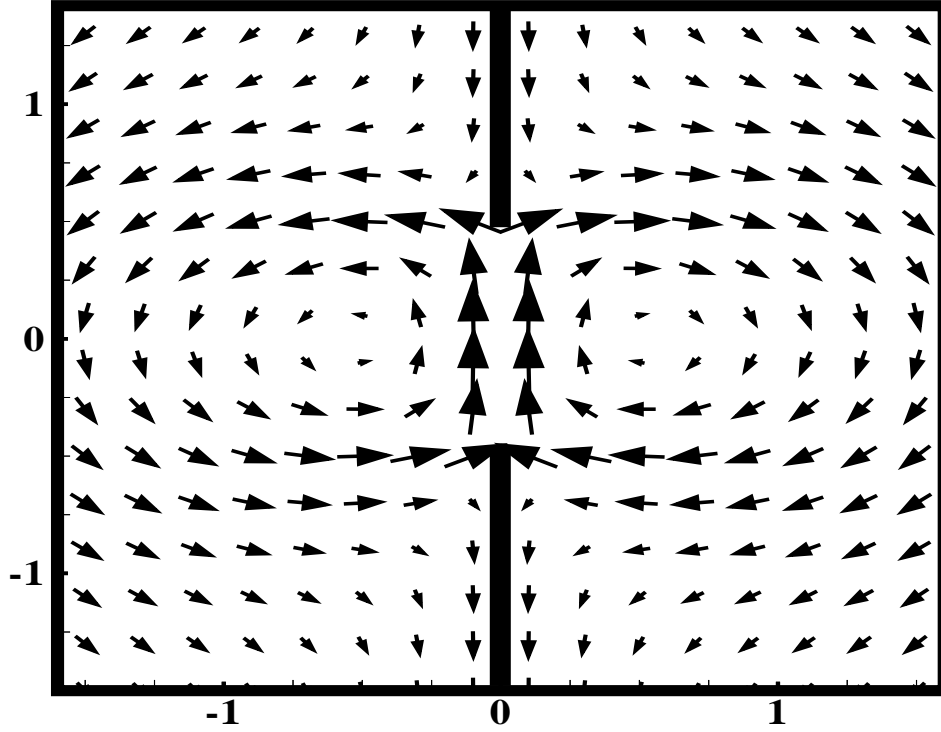
Figure 6. Vector plot of the current for $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$, $\phi = \pi$, $\alpha_l = \alpha_r = 0$. Axes are marked in the units of a. Because of our non-self-consistent formalism $\xi_0 = 5a$. It is similar to the s-wave junction in [22].

6) The parallel current, $\mathbf{j}_y$, is plotted in terms of the phase difference for different super-fluid velocities and at $\alpha_l = \alpha_r = 0$, the maximum value of current and appearance of vortex-like currents occur at $\phi = \pi$. In this case, far from $\phi = \pi$, we observe a constant current that is the external transport current on the banks (Fig.2).

7) For $\alpha_{l,r} = 0$, $\alpha_{r,l} = \pm\frac{\pi}{4}$ the maximum values of the parallel current, $\mathbf{j}_y$, and consequently the vortices appear at $\phi = 0$, $\phi = \pi$, $\phi = 2\pi$ (Figs.4,5).

8) For $\alpha_{l,r} = 0$, $\alpha_{r,l} = \frac{\pi}{2}$ the current-phase graphs are similar to the $\alpha_l = \alpha_r = 0$ but a displacement as much as π occurs (Figs.2,3). Thus the vortices can be observed at $\phi = 0$ and $\phi = 2\pi$. But at $\phi = \pi$, we do not observe the vortex-like currents. This can be another difference between conventional and unconventional Josephson junctions.

9) The superposition of dash lines in Figs.4,5 for zero super-fluid velocity, and all the lines of Fig.2, for zero mis-orientations gives us the three lines of Figs.4,5 apparently. This means that in this case the tangential current is algebraic sum of transport current, "interference" current[22] and spontaneous current [6].

10) The tangential current for $\alpha_l = \alpha_r = 0$ and $\alpha_{l,r} = 0, \alpha_{r,l} = \frac{\pi}{2}$ is an even function of Josephson phase ϕ but for $\alpha_{l,r} = 0, \alpha_{r,l} = \pm\frac{\pi}{4}$ it is neither even nor odd function of
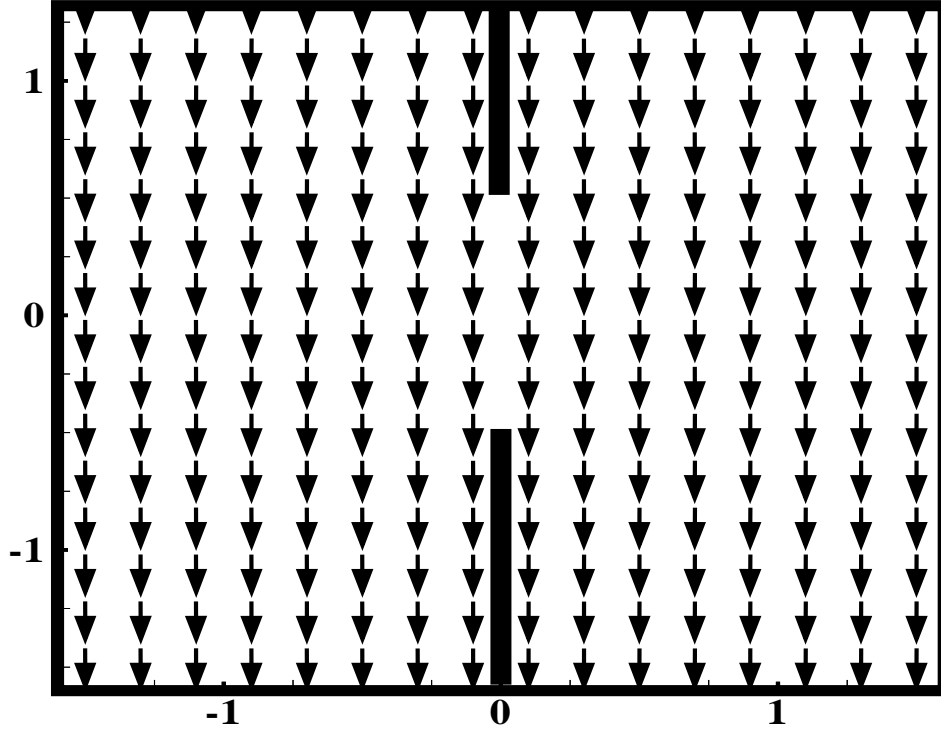
Figure 7. Vector plot of the current for $\phi = 0$, $\alpha_l = 0$, $\alpha_r = 0$ and $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$. Vortices disappeared but transport supercurrent flows.

phase difference ϕ and the symmetry will be broken.

The superfluid of pairs creates the transport current but the spontaneous current is produced by mis-orientation plus to the phase difference. However the "interference" current depends on all of the parameters (phase difference, mis-orientation and superfluid velocity) and it can be the result of the non-locality of supercurrent.

The spatial distributions of the order parameter and the current near and precisely at the contact are calculated using the Green function along the transit and non-transit trajectories numerically. Transit trajectories for each point are coming from the orifice (transparent part of interface $x = 0, |y| \leq a$) while non-transit trajectories form the remainder part of interface which is impenetrable (reflective part of interface $x = 0, |y| \geq a$). The current distributions are calculated and simulated numerically for $T/T_c = 0.1$, $p_F v_s/\Delta_0(0) = 0.5$ and different choices of mis-orientation for $\phi = \pi$ (Figs.8,6,9) and $\phi = 0$ (Fig.7). Following the Ref.[22], the "interference" current that plays a central role for production of vortex-like current is found. At the phase values $0 < \phi < \frac{\pi}{2}$, $\frac{3}{2}\pi < \phi < 2\pi$ and $\alpha_l = \alpha_r = 0$, the interference current is very small thus the total current is close to the vector sum of transport, spontaneous and Josephson currents. But, for $\frac{\pi}{2} < \phi < \frac{3}{2}\pi$ the "interference" current appears and the total current
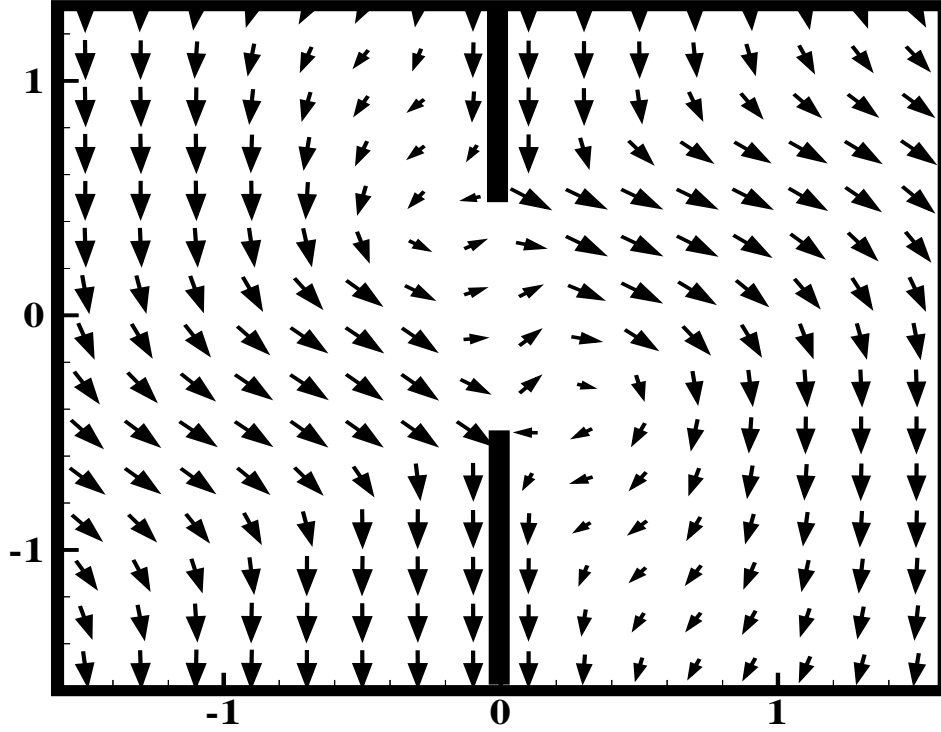
Figure 8. Vector plot of the current for $\phi = \pi$, $\alpha_l = 0$, $\alpha_r = \frac{\pi}{4}$ and $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$. Axes are marked in the units of a.

deviates from the vector sum of the three old currents [22]. Also, for zero mis-orientation the spontaneous current is zero.

The "interference" current is always anti-parallel to the transport current. But the spontaneous current may be parallel or anti-parallel to the transport current. If algebraic sum of transport, "interference" and spontaneous current is anti-parallel to the transport current on the banks, we can find the vortex-like currents (Fig.6).

Thus the appearance of the vortex-like currents can be controlled by mis-orientation and phase difference. It is remarkable that, far from the contact $|\mathbf{r}| \sim \xi_0 > a$ for all ϕs, mis-orientations, temperatures T and superfluid velocities, the distributions of currents tend to the tangential transport currents on the banks.

4. near the critical temperature

For temperatures close to the critical temperature, $T_c - T \ll T_c$ problem is solvable pure analytically [22]. At the contact we have

$$\mathbf{j} = \mathbf{j}_J + \mathbf{j}_{spont} + \mathbf{j}_T + \mathbf{j}_{JT} \tag{12}$$
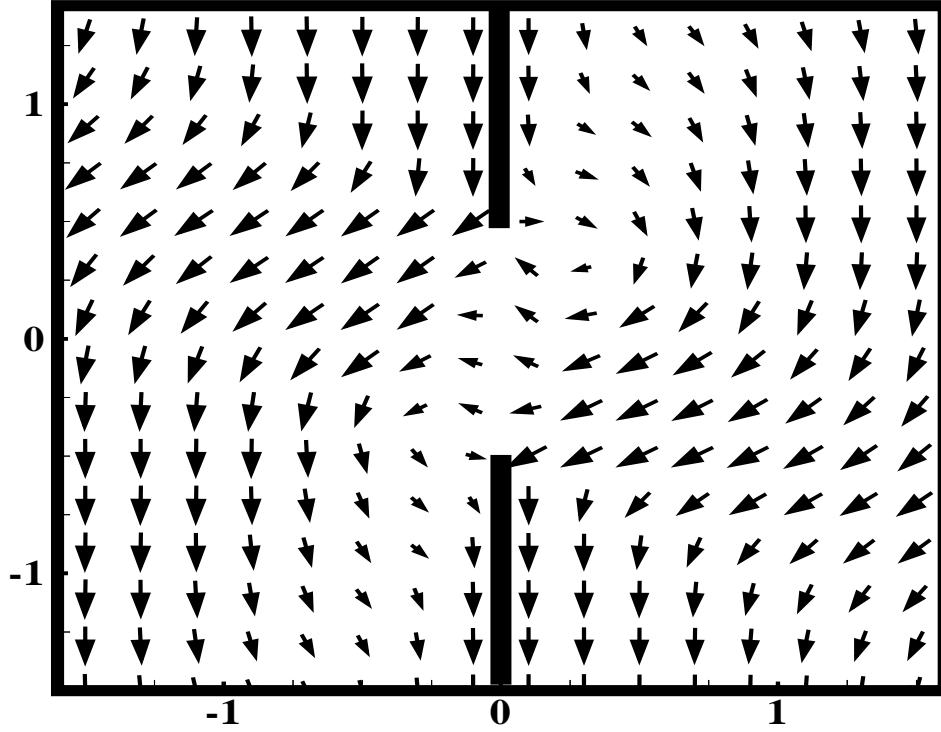
Figure 9. Vector plot of the current for $\phi = \pi$, $\alpha_l = 0$, $\alpha_r = -\frac{\pi}{4}$ and $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$. The axis of vortices is rotated.

$$\mathbf{j}_J = 2j_c \sin\phi \left\langle \widehat{\mathbf{v}}_x sign(v_x)(\frac{\Delta_l \Delta_r}{\Delta_0^2}) \right\rangle_{\widehat{\mathbf{v}}} \tag{13}$$

$$\mathbf{j}_{spont} = 2j_c \sin\phi \left\langle \widehat{\mathbf{v}}_y sign(v_x)(\frac{\Delta_l \Delta_r}{\Delta_0^2}) \right\rangle_{\widehat{\mathbf{v}}} \tag{14}$$

$$\mathbf{j}_T = -j_c k \left\langle \widehat{\mathbf{v}} \widehat{v}_y (\frac{\Delta_l \Delta_r}{\Delta_0^2}) \right\rangle_{\widehat{\mathbf{v}}} \tag{15}$$

$$\mathbf{j}_{JT} = j_c k(1 - \cos\phi) \left\langle \widehat{\mathbf{v}} \widehat{v}_y (\frac{\Delta_l \Delta_r}{\Delta_0^2}) \right\rangle_{\widehat{\mathbf{v}}} \tag{16}$$

where, as in Ref.[22]

$$j_c(T, v_s) = \frac{\pi |e| N(0) v_F}{8} \frac{\Delta_0^2(T, v_s)}{T_c} \tag{17}$$

is a critical current of the contact at $(T_c - T) \ll T_c$, k is a standard notation

$$k = (14\varsigma(3)/\pi^3)(v_s p_F/T_c). \tag{18}$$

For the high value of temperatures near the T_c, the critical values of currents have a linear dependence on the Δ_0^2, which can be replaced by $\Delta_0 = \sqrt{\left(\frac{32\pi^2}{21\zeta(3)}\right) T_c(T_c - T)}$.

On the other hand the spontaneous and Josephson currents are sinusoidal function of phase difference as is expected for the currents near the T_c, in spite of these two terms of current the "interference" current is an even function of the phase difference. The current is divided into the four parts, Josephson current $\mathbf{j}_J$, spontaneous current, transport current in the banks $\mathbf{j}_T$ and the "interference" current $\mathbf{j}_{JT}$. It is observed that, the currents generally and near the T_c obviously, depend not only on the mis-orientation $|\alpha_l - \alpha_r|$ but also depend on the orientations with respect to the interface. Because in the expressions $\langle \hat{\mathbf{v}} \Delta_l \Delta_r ... \rangle$, the result of angular integrations may include both $|\alpha_l - \alpha_r|$ and $|\alpha_l + \alpha_r|$ terms. For example, for Josephson and spontaneous currents in (13) and (14) by angular integration on the Fermi surface we have

$$\mathbf{j}_J = (\frac{2j_c \sin\phi}{15\pi})[15\cos(2\alpha_l - 2\alpha_r) - \cos(2\alpha_l + 2\alpha_r)]\hat{\mathbf{x}} \tag{19}$$

and for spontaneous current

$$\mathbf{j}_{spont} = (\frac{-8j_c \sin\phi}{15\pi}) \sin(2\alpha_l + 2\alpha_r)\hat{\mathbf{y}} \tag{20}$$

respectively. Also it is obtained that for $\phi = \pi$ and exactly at the contact, the "interference" current $\mathbf{j}_{JT}$ is anti-parallel to the $\mathbf{j}_T$. For $\phi = \pi$ the "interference", Josephson and spontaneous currents are $\mathbf{j}_{JT} = -2\mathbf{j}_T$, $\mathbf{j}_J = 0$ and $\mathbf{j}_{spont} = 0$ respectively, consequently it is obtained that $\mathbf{j}_y = \mathbf{j}_T + \mathbf{j}_{JT} + \mathbf{j}_{spont} = -\mathbf{j}_T$. In this case while the Josephson current is zero the terms $\mathbf{j}_y$ and $\mathbf{j}_T$ that are directed opposite to the each other, control the appearance of vortex-like currents in vicinity of the point contact. In addition, for $\phi = 0$, $\alpha_{l,r} = 0$ and $\alpha_{r,l} = \frac{\pi}{2}$, we can observe the vortex-like currents. This property can be a difference between d-wave and s-wave Josephson junctions. Because in the s-wave Josephson junction the vortex-like currents are observed only for $\phi = \pi$ (Ref.[22]), while in the present calculations for d-wave Josephson junction, the vortex-like currents may appear even for $\phi = 0$. Thus, for fixed values of the temperature and superfluid velocity, the presence of vortex-like currents can be controlled by mis-orientation and phase difference.

5. Conclusions

The vortex-like currents in vicinity of the point contact are observed for d-wave Josephson junction as well as for s-wave junction in Ref.[22]. The interference current as a result of non-local supercurrent states, appears. It may flow opposite to the external transport current. For $\phi = \pi$, $\alpha_l = 0$ and $\alpha_r = \pm\frac{\pi}{4}$ the vortex-like currents with the rotated axis are observed. But as is obtained in Ref.[22], the axis of vortex-like currents in the s-wave Josephson junction is normal to the interface. Thus, this rotated axis can be used to distinguish between s-wave and d-wave junction. Also it can be exerted to distinguish between the junction between two pure s-wave and mixing of conventional and unconventional order parameters (eg.$d + is$). In addition to the ZES, this rotated axis can be another "fingerprint" of d-wave pairing symmetry. Another

interesting result is, the behavior of system in the absence of the phase difference. For s-wave system only in the presence of the phase difference, $\phi \simeq \pi \neq 0$, the vortex-like currents appear [22], while for d-wave Josephson junction at zero phase and zero external magnetic flux, it is possible to observe the vortex-like currents for some mis-orientations. This can be a theoretical reason that, the mis-orientation plays role instead of the phase difference (Josephson phase). In the stationary Josephson junction $\mathbf{v_s} = \mathbf{0}$ the tangential interference current (spontaneous current) will be observed only for $\phi \neq 0$ but in the opposite case $\mathbf{v_s} \neq \mathbf{0}$ the tangential current even in the absence of phase difference may be observed. In addition, this tangential current can flow in the opposite direction to the external transport current and this factor can produce the vortex-like currents. Finally, playing the role of magnetic Josephson phase ($\phi = \frac{q\Phi}{\hbar}$ where q and Φ are electric charge and magnetic flux respectively) by mis-orientation of superconducting ab-planes (pairing symmetry in the momentum space) is a reason for magnetic nature of pairing mechanism in the high T_c superconductors which remains as an unknown and famous problem.

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